Session 8: Energy, Work, Power & Contact Mechanics

Part A- Energy, Work, and Power

Part B: Friction, Contact, and Compliance

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Part A: Energy, Work, and Power

The Physics Behind Robot Motion: Energy, Work & Power

- Robots **move**, **lift**, **rotate**, **balance**, and **manipulate** objects.
- Every motion requires **energy** consumption.
- Understanding energy, work, and power is essential for:
  - Robot design and hardware selection
  - Actuator sizing and control
  - Battery life and power optimization
  - Motion planning and task scheduling
  - Ensuring safety, stability, efficiency
- These are **universal principles** governing all robotic systems.

What We'll Study Today

Key Questions

- How do robots convert energy into motion?
- How is work calculated in robotic systems?
- How much power is required to perform tasks?
- How do energy concepts influence robotic performance?

Key concepts:

- **Energy** – Capacity to perform work.
- **Work** – Energy transferred via force acting over distance.
- **Power** – Rate at which work is performed.
- **Kinetic Energy** – Energy of motion.
- **Potential Energy** – Stored energy due to position.

Approach:

- First: Build the physics foundation.
- Then: Apply it to real robotic systems.

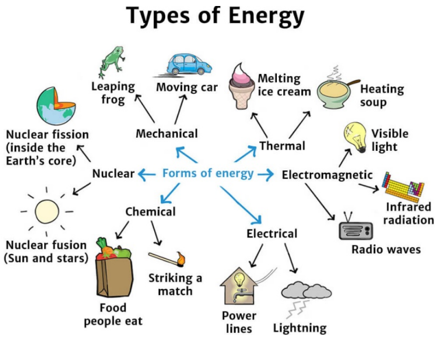
Energy — The Capacity to Do Work

Energy is defined as the measure of the ability to do work.

- Energy determines the capacity of a system to perform work and may be stored or transferred in various forms.
- Energy is a scalar quantity.
- SI Unit of Energy: Joule (J):

$$1\,\mathrm{J} = 1\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2$$

- Energy can exist in various forms:
  - Mechanical Energy (Kinetic, Potential)
  - Chemical Energy
  - Electrical Energy
  - Nuclear Energy
  - Electromagnetic Energy
  - Acoustic Energy
  - Thermal Energy



Mechanical Energy Focus (for Robotics)

- In this session, we will mainly focus on **mechanical forms of energy** (Kinetic and Potential).
- The same basic principles apply to all forms, but our attention is on robotics-relevant energy.
- **Mechanical Energy in Robotics:**
  - **Kinetic Energy**: due to motion (robot arms, wheels, mobile robots).
  - **Potential Energy**: due to position (height, spring compression, configuration).
- **Energy Storage vs Transfer:**
  - **Stored Energy**: energy contained in position or motion (raised arm, spinning flywheel).
  - **Transient Energy**: energy being transferred between components (motors, gears).

Kinetic Energy

- **Definition:** Kinetic energy is the energy possessed by an object due to its motion. The faster an object moves, or the more massive it is, the more kinetic energy it has.
- **Mathematical Definition:** $K = \frac{1}{2}mv^2$
- **Scalar Quantity:** Always positive, no direction.
- **SI Unit:** Joule (J), $1\,J = 1\,\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$
- **Other Units:** $1\,\mathrm{erg} = 10^{-7}\,J; \quad 1\,\mathrm{eV} = 1.60 \times 10^{-19}\,J$

Robotics Applications:

- Translational motion of mobile robots.
- Moving arms, links, and payloads.
- Both translational and rotational kinetic energy are involved in manipulator dynamics.
- Helps in actuator sizing, motion planning, and energy budgeting.

Work — Definition

- Work is done when a force causes displacement.
- Work measures energy transfer into or out of a system due to applied forces.
- No displacement $\implies$ no work done.

Work Done by Constant Force:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \phi$$

Where ϕ is the angle between force and displacement.

Interpretation of ϕ:

- $\phi = 0^\circ$: maximum positive work
- $\phi = 90^\circ$: zero work
- $\phi = 180^\circ$: negative work

Units of Work:

$$1\,\mathrm{J} = 1\,\mathrm{N} \cdot \mathrm{m} = 1\,\frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2}$$

Work — Net Work and Variable Forces

Net Work from Multiple Forces:

$$W_{\text{net}} = \sum(\vec{F} \cdot \vec{d}) = \vec{F}_{\text{net}} \cdot \vec{d}$$

Work by Variable Force — 1D Case:

$$W = \int_{x_i}^{x_f} F_x(x)\, dx$$

General Work for 3D Case:

$$W = \int_{x_i}^{x_f} F_x(r) dx + \int_{y_i}^{y_f} F_y(r) dy + \int_{z_i}^{z_f} F_z(r) dz or compactly : W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

Work — Special Case: Gravity & Robotics Applications

Work Done by Gravity (Vertical Displacement):

$$W_{\text{grav}} = -mg\Delta y$$

Robotics Applications of Work:

- Work done while lifting payloads with robot arms.
- Work done during 3D motion of manipulators.
- Work done during multi-joint coordinated movements.

Spring Force (Hooke's Law)

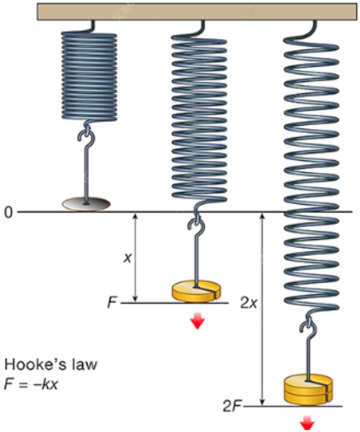
- Springs exert restoring forces when stretched or compressed.
- The force depends directly on displacement from equilibrium.

Hooke's Law:

$$F_x = -kx$$

Where:

- F_x = restoring force
- k = spring constant (N/m)
- x = displacement from equilibrium
- The negative sign: force opposes displacement direction.



Work Done by Spring Force

Work done by the spring from x_i to x_f:

$$W_{\text{spring}} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

- Work depends only on initial and final positions.
- Characteristic of conservative forces.

Robotics Applications of Spring Forces:

- Compliant actuators, flexible joints.
- Soft robotics, force sensors.
- Energy-efficient compliant mechanisms.

Work-Energy Theorem

What happens to the work done on a system?

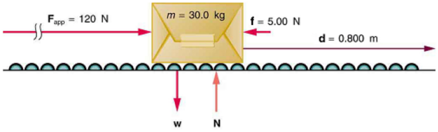
- Energy is transferred into the system, but in what form?
- Often, work goes into changing kinetic energy.

Example (Roller Conveyor System):

- Package accelerated horizontally.
- Vertical forces cancel (no work done by gravity/normal).
- Only horizontal net force does work.

Net Work:

$$W_{\text{net}} = F_{\text{net}} d$$



Using Newton's Second Law:

$$F_{\text{net}} = ma \quad \Rightarrow \quad W_{\text{net}} = mad$$

Work-Energy Theorem

Relating Work to Change in Speed:

- Kinematic relation: $v^2 = v_0^2 + 2ad$
- Solve for a:

$$a = \frac{v^2 - v_0^2}{2d}$$

Substitute into work equation:

$$W_{\text{net}} = m\left(\frac{v^2 - v_0^2}{2d}\right)d$$

Simplifying:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad \text{(Eq. 3.3.4)}$$

Work-Energy Theorem

Work-Energy Theorem (General Form):

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad \text{(Eq. 3.3.5)}$$

or simply:

$$W_{\text{net}} = \Delta K$$

Key Robotics Insight:

- In robotics, actuators do work to change the robot's kinetic energy.
- Helps estimate energy requirements during acceleration, deceleration, and motion planning.

Power

- Power is the rate at which work is done.
- It tells how fast energy is being transferred or transformed.

Average Power:

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad \text{(Eq. 6.14)}$$

Instantaneous Power:

$$P = \frac{dW}{dt} \quad \text{(Eq. 6.15)}$$

Units of Power:

$$1\,\text{W} = 1\,\frac{\text{J}}{\text{s}} = 1\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}^3} \quad \text{(Eq. 6.16)}$$

Power

Power in terms of Force and Velocity:

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi \quad \text{(Eq. 6.17)}$$

Robotics Application:

- Determines actuator energy consumption per second.
- Important for motor sizing, battery capacity, and thermal design.
- Helps analyze both continuous and peak power demands.

Conservative Forces

- A force is **conservative** if the work it does depends only on the initial and final positions, not on the path taken.
- **Path Independence:**

$$W_{\text{closed path}} = 0$$

- Energy can be fully recovered when motion is reversed.

Examples of Conservative Forces:

- Gravitational Force
- Spring Force (Hooke's Law)
- Electrostatic Force

Non-Conservative Forces (for contrast):

- Friction
- Air Resistance
- Damping Forces

Importance in Robotics:

- Simplifies energy calculations (energy conservation).
- Allows energy storage (springs, counterweights, gravity balancing arms).
- Enables energy-efficient design for compliant and assistive robots.

Potential Energy: Concept & Conservative Forces

- Work done by conservative forces equals negative change in potential energy:

$$W_{r_i \to r_f} = -\Delta U = U(r_i) - U(r_f)$$

- r_i = initial position
- r_f = final position

When to use Potential Energy:

- Conservative forces: use potential energy change.
- Non-conservative forces: calculate work directly.

Conservative Forces Encountered So Far:

- Gravity
- Spring Force (Hooke's Law)

Forms of Potential Energy

Gravitational Potential Energy:

- The simplest conservative force is gravity near Earth's surface:

$$\vec{F}_{\text{gravity}} = -mg\hat{j}$$

- Corresponding potential energy:

$$U_{\text{grav}} = mgy$$

- SI Units: $[U] = \text{J} = \text{kg} \cdot \text{m}^2/\text{s}^2$

Spring (Elastic) Potential Energy:

- A spring with force constant k extended by x stores:

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

- SI Units: $[U] = \text{J}$

Key Idea: Potential energy stores mechanical energy that can convert to kinetic energy.

Robotics Applications of Potential Energy

- Gravity compensation using counterweights, springs, and balancing arms.
- Energy storage in compliant actuators (e.g. Series Elastic Actuators).
- Smooth energy recovery in walking robots, prosthetics, exoskeletons.
- Simplifies robot dynamics using conservation of mechanical energy.

Conservation of Mechanical Energy

Work-Kinetic Energy Theorem (including conservative and non-conservative forces):

$$W = W_{\text{cons}} + W_{\text{non-cons}} = \Delta K$$

From conservative forces:

$$W_{\text{cons}} = -\Delta U$$

Substituting gives:

$$-\Delta U + W_{\text{non-cons}} = \Delta K$$

Rearranged form:

$$\Delta K + \Delta U = W_{\text{non-cons}}$$

Total energy is defined as:

$$E = K + U$$

Thus:

$$\Delta E = \Delta K + \Delta U = W_{\text{non-cons}}$$

Conservation of Mechanical Energy (Special Case)

The total mechanical energy changes by the work done by non-conservative forces.
In many problems, all forces are conservative (negligible friction, air resistance), so:

$$W_{\text{non-cons}} = 0$$

Thus:

$$\Delta E = \Delta K + \Delta U = 0$$

or equivalently:

$$K_i + U_i = K_f + U_f$$

$$E_i = E_f$$

In these cases, total mechanical energy remains constant.
Energy shifts between kinetic and potential forms, but total energy stays the same.

Rotational Work and Energy: The Grindstone Example

- Work must be done to rotate objects like grindstones or merry-go-rounds.
- A motor spins the grindstone. Sparks, noise, and vibration are created while steel layers are removed.
- After the motor turns off, the grindstone keeps rotating, but gradually slows down due to friction.

The work done by the motor is converted into:

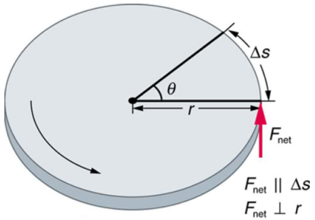
- Rotational kinetic energy
- Heat, Light, Sound
- Vibration

Rotational Work: Concept and Definition

Understanding Rotational Work

- Work must be done to rotate objects such as grindstones or merry-go-rounds.
- We build on the knowledge of translational work to analyze rotational work.
- In the simplest rotational case, the net force is applied perpendicular to the radius of the disk and remains perpendicular as the disk rotates.
- Since the force is parallel to the displacement, the work done is:

$$W_{net} = (\mathrm{F}_{net})\Delta s$$



Rotational Work: Deriving Torque Relation

- Multiply and divide by r:

$$W_{net} = (r \cdot \mathrm{F}_{net}) \frac{\Delta s}{r}$$

- Recognize:

$$r \cdot \mathrm{F}_{net} = \tau_{\text{net}} \quad \text{and} \quad \frac{\Delta s}{r} = \theta$$

- Substituting gives:

$$W_{net} = \tau_{\text{net}} \cdot \theta$$

Key Understanding:

- Torque τ plays the same role in rotation that force F plays in translation.
- Angular displacement θ corresponds to linear displacement Δs.

Rotational Work-Energy Theorem

Start from Rotational Kinematics:

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

This relates:

- ω: angular velocity
- ω_0: initial angular velocity
- α: angular acceleration
- θ: angular displacement

Rearrange to isolate $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}$$

Rotational Work: Deriving Torque Relation

Substitute into Work Expression:

$$W_{net} = I\alpha\theta$$

Substituting:

$$W_{net} = I\left(\frac{\omega^2 - \omega_0^2}{2}\right)$$

Simplifying:

$$W_{net} = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$$

Key Understanding:
The work done by net torque changes the rotational kinetic energy. This is the rotational form of the work-energy theorem.

Rotational Kinetic Energy

Definition of Rotational Kinetic Energy:

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

Where:

- I: moment of inertia
- ω: angular velocity

Comparison to Translational Kinetic Energy:

$$KE_{\text{trans}} = \frac{1}{2} m v^2$$

$$m \leftrightarrow I, \quad v \leftrightarrow \omega$$

Key Idea:
Rotating objects store energy in their rotation, just like moving objects store energy in translation.

Total Mechanical Energy in Rotational Systems

Total Mechanical Energy:

$$E_{\text{total}} = KE_{\text{trans}} + KE_{\text{rot}} + U$$

Where:

- $KE_{\text{trans}} = \frac{1}{2}mv^2 \quad$ (Translational kinetic energy)
- $KE_{\text{rot}} = \frac{1}{2}I\omega^2 \quad$ (Rotational kinetic energy)
- $U \quad$ (Potential energy)

Energy Conservation:

- Total mechanical energy remains constant if only conservative forces act.
- Energy can shift between translation, rotation, and potential forms.

How Thick Is the Soup? Rolling and Energy Division (Part 1)

Rolling Motion and Energy Conservation:

- Cans roll down a ramp starting from rest.

Initial gravitational potential energy:

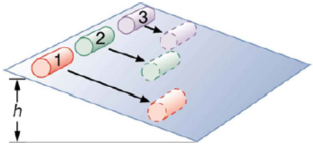
$$PE_{\text{initial}} = mgh$$

Energy divides into translation and rotation:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Explanation:

- The initial potential energy is divided between translational and rotational kinetic energy.



How Thick Is the Soup? Rolling and Energy Division (Part 2)

- The greater the moment of inertia I, the less energy goes into translation.
- If the can slides down without friction (no rotation), all energy goes into translation.
- Therefore, the can moves faster when not rotating.

Key Understanding:

- Rotation reduces translational speed because part of the potential energy is stored as rotational kinetic energy.
- Thicker soup inside the can rotates with it, increasing I, and reducing translation speed further.

Part B: Energy, Work & Power in Robotics Actuation- Arduino Motor Demo Lab

Why Are We Doing This?

- Robots consume energy to move.
- Energy converts from battery (electrical) to motor movement (mechanical).
- We need to understand:
  - How much energy is used?
  - How power depends on speed and load.
- Today: We simulate and measure this.

Project Objective

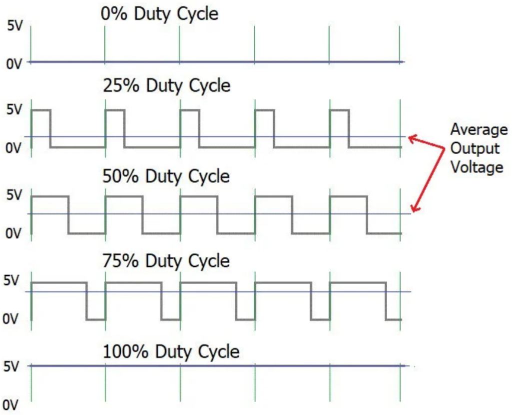
- Convert electrical energy into mechanical work.
- Control motor speed via PWM.
- Measure: Voltage, Current, RPM.
- Calculate:
  - Power: $P = V \times I$
  - Work: $W = P \times t$
  - Efficiency
- Study how load and speed affect power draw.

Work, Power & Energy Recap

- Work: $W = F \times d$ (or Torque $\times$ Angle for motors)
- Power: $P = \frac{W}{t}$ or $P = V \times I$
- Power tells how fast energy is consumed.

How We Control Power in Motors: PWM

- Arduino can't output analog voltage.
- PWM = fast ON-OFF switching.
- Duty Cycle controls average voltage.
- More ON time $\rightarrow$ More Power $\rightarrow$ Faster motor.



What is Tinkercad? (Our Virtual Lab)

- Online simulator for electronics.
- Build circuits virtually.
- Connect: Arduino, Motor Driver, DC Motor, Battery, Multimeters.
- Write real Arduino code.
- Measure: Voltage, Current, RPM.
- Link: `https://www.tinkercad.com/`

Hardware Setup

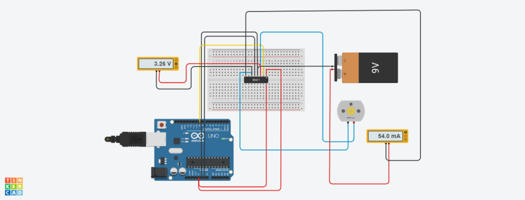
- Arduino UNO
- L293D Motor Driver
- DC Motor
- 9V Battery
- 2 Multimeters
- Breadboard

Quick Pin Mapping Summary

- Pin 1 (Enable 1) $\rightarrow$ Arduino 5V
- Pin 2 (Input 1) $\rightarrow$ Arduino Pin 9
- Pin 3 (Output 1) $\rightarrow$ Motor terminal 1 (+ Voltmeter)
- Pins 4,5,12,13 $\rightarrow$ GND
- Pin 6 (Output 2) $\rightarrow$ Motor terminal 2 (- Voltmeter)
- Pin 7 (Input 2) $\rightarrow$ GND
- Pin 8 (Vcc2) $\rightarrow$ Ammeter Black probe
- Pin 16 (Vcc1) $\rightarrow$ Arduino 5V
- Ammeter Red probe $\rightarrow$ Battery +
- Battery - $\rightarrow$ GND Rail

Circuit Diagram

- Full wiring of:
  - Arduino
  - L293D Driver
  - Motor
  - Battery
  - Multimeters



Arduino Code

```
int motorPin = 9;
void setup() { pinMode(motorPin, OUTPUT); }
void loop() { analogWrite(motorPin, PWM_VALUE); delay(3000); }
```

What We Will Measure

- Voltage (V)
- Current (A)
- Motor speed (RPM)
- Calculate:
  - Power: $P = V \times I$
  - Work: $W = P \times t$

Live Demo Steps

1. Upload code and run motor.
2. Measure V, I, RPM.
3. Calculate power and work.
4. Change PWM value.
5. Repeat and fill data table.

Student Observation Table

PWM	V	I	RPM	Power	Work
255					
200					
153					
102					

Load vs Current

- More load $\rightarrow$ More torque needed.
- Torque $\propto$ Current.
- More current $\rightarrow$ More power drawn.
- Real robots must budget current carefully.

Real-World Relevance

- Power management affects:
  - Battery life
  - Robot weight
  - Heat generation
  - Efficiency
- Used in:
  - Drones
  - Autonomous cars
  - Robot arms
  - Mobile robots

Summary & Takeaways

- PWM controls motor speed.
- Power & Work can be calculated from measurements.
- Load affects current draw.
- In robotics: Every milliamp matters!