Session 6: Free body Diagrams, COM and Dynamics

Part A- Free Body diagrams
Part B- COM
Part C: Dynamics

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Session 5 Recap & Reflect: What Did We Learn?

Torque – Basics and Types

- **What is Torque?**
  - Torque is a measure of the **rotational effect** of a force.
  - It determines how effectively a force can cause rotation about an axis.
  - **Formula:**

$$\tau = r \times F = rF\sin(\theta)$$

  - r = distance from axis of rotation (lever arm)
    F = applied force
    θ = angle between $\vec{r}$ and $\vec{F}$

- **Types of Torque**
  - **Static Torque:** Torque applied but **no rotation** occurs.
    - E.g., Holding a door still without moving it.
  - **Dynamic Torque:** Torque that causes **angular acceleration**.
    - E.g., A motor spinning a fan or car wheels accelerating.

Session 5: Recap & Reflect (contd.)

Newton's Second Law for Rotation

- In linear motion: $F = m \cdot a$
- In rotational motion: $\tau_{\text{net}} = I \cdot \alpha$
- Where:
  - τ_{net}: Net torque
  - I: Moment of inertia (resistance to rotation)
  - α: Angular acceleration
- **Moment of inertia** depends on mass distribution — the farther mass is from the axis, the greater the inertia.

In Robotics:

- Calculate joint torques for robotic arms
- Estimate motor response time
- Determine energy needs for drone maneuvers

Session 5 Recap & Reflect

Torque Analysis (Balanced vs. Unbalanced)

- **Balanced Torque**
  - Net torque is zero: $\sum \tau = 0$
  - No angular acceleration; system remains stable.
  - Seen in: balanced seesaw, symmetric lever systems.
  - Follows the Principle of Moments: $\sum \tau_{\text{cw}} = \sum \tau_{\text{ccw}}$

- **Unbalanced Torque**
  - Net torque is non-zero: $\sum \tau \neq 0$
  - Results in angular acceleration; system rotates.
  - Caused by unequal forces or distances from pivot.

Static & Dynamic Equilibrium

- **What is Equilibrium?**
  - A system is in equilibrium if the net force and net torque on it are zero.
  - This means: no linear or angular acceleration.
  - Equilibrium is essential for robotic balance, holding, and precise motion.

Static Equilibrium

- Object is at rest.
- $\sum \vec{F} = 0$ and $\sum \tau = 0$
- No motion or rotation.
- *Examples:*
  - Book resting on a table
  - Balanced seesaw
  - Stationary robotic arm

Dynamic Equilibrium

- Object in constant motion.
- $\sum \vec{F} = 0$ and $\sum \tau = 0$
- No acceleration despite movement.
- *Examples:*
  - Car moving at constant velocity
  - Drone flying steadily
  - Skydiver at terminal velocity

Stability Types

What Happens When a System in Equilibrium is Disturbed?

- **Stable Equilibrium:**
  - Returns to its original position when disturbed.
  - A restoring force or torque opposes the displacement.
  - *Example:* A pendulum at the bottom position.
- **Unstable Equilibrium:**
  - Moves further away when disturbed.
  - Disturbing force or torque acts in the same direction.
  - *Example:* Pencil balanced on its tip.
- **Neutral Equilibrium:**
  - Stays in the new position after being disturbed.
  - No net force or torque tries to restore or disturb further.
  - *Example:* Ball on a flat table.

Recap Quiz

Q1. A robot with most of its mass located near the top is gently pushed. It starts to tip and, after it's released, the tipping continues without returning. What does this indicate?

A. The robot was in stable equilibrium but temporarily unbalanced
B. The robot is in neutral equilibrium and will stay wherever it stops
C. The robot was in unstable equilibrium and experiences increasing torque away from its position
D. The robot's base has no torque acting on it

Q2. A robot arm applies a constant forward force to an object on a frictionless table. The object continuously accelerates as long as the force is applied. What does this tell us about the force balance?

A. The object is in **dynamic equilibrium** — moving at constant speed
B. The forces are **balanced** — net force is zero
C. The applied force is **unbalanced** — net force causes acceleration
D. There is **no net force** — motion will stop once the force is removed

Recap Quiz (contd.)

Q3. A conveyor belt moves at constant speed carrying boxes. A robotic arm remains stationary next to it, ready to pick items. Which parts of the system are in dynamic and static equilibrium, respectively?

A. Belt – Static; Arm – Dynamic
B. Belt – Dynamic; Arm – Static
C. Both are in dynamic equilibrium
D. Belt is not in equilibrium; Arm is in dynamic equilibrium

Q4. A beam is pivoted at its center. Equal weights are hung at equal distances on both ends, but one is suspended vertically, and the other via a pulley that redirects the force horizontally. Which statement is true?

A. Both forces create equal torques in opposite directions
B. Only the vertically hanging weight causes torque
C. The horizontal force produces zero torque
D. The torques cancel only if the mass is doubled on the horizontal side

Recap Quiz (contd.)

Q5. Two robots with arms of equal length lift the same weight. One robot's arm has most of its mass near the shoulder, while the other's is spread to the wrist. Who needs more torque to lift the same weight?

A. The one with mass near the shoulder
B. The one with mass near the wrist
C. Both need the same torque
D. Depends only on the weight being lifted

Q6. A quadcopter suddenly accelerates clockwise in yaw. Which of the following could be a valid explanation?

A. All rotors are spinning at the same speed
B. Two clockwise rotors are spinning faster than the counter-clockwise ones
C. Gravity is unbalanced
D. The quadrotor is in static equilibrium

Answer Key with Explanations

Q1. C – High center of mass leads to tipping; torque increases as it rotates.

Q2. C – On a frictionless surface, continuous acceleration indicates a **net unbalanced force**. According to Newton's Second Law, $F = ma$, this unbalanced force causes the object to accelerate.

Q3. B – Belt is moving at constant speed (dynamic); arm is still (static).

Q4. C – Horizontal force produces zero torque (angle $= 0^\circ \rightarrow \sin(0) = 0$).

Q5. B – Greater moment of inertia with mass farther from pivot $\rightarrow$ needs more torque.

Q6. B – Uneven rotor speeds cause yaw; clockwise torque dominates.

Part A- Free-Body Diagram (FBD)

What is a Free-Body Diagram (FBD)?

- **Definition:**
  - A **Free-Body Diagram (FBD)** is a simplified sketch that shows all the **external forces and torques** acting on a single object.
  - It helps isolate the object from its environment, focusing only on what influences its motion or balance.
  - Think of it as: *"Zooming in on one object to analyze what's pushing or pulling on it."*

- **Why Use FBDs?**
  - Clarifies complex mechanical situations by simplifying them.
  - Helps apply Newton's Laws ($\sum \vec{F} = 0$, $\sum \tau = 0$, or $\sum \vec{F} = m\vec{a}$).
  - Essential for solving equilibrium and dynamics problems.
  - Helps identify incorrect assumptions or missing forces.
  - Bridges the gap between physical intuition and mathematical formulation.

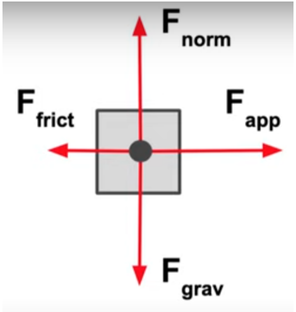
FBDs in Robotics

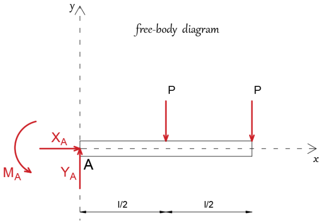
- **Why FBDs Matter in Robotics:**
  - FBDs help calculate the **torque** required at joints to lift, hold, or move objects.
  - They aid in identifying **external forces** such as gravity, payloads, or ground reaction forces.
- **Without FBDs:**
  - Force and torque calculations become guesswork.
  - Increased risk of incorrect actuator sizing or overload.

Features of FBD

- A Free-Body Diagram (FBD) shows all external forces acting on a single object.
- It helps predict how an object will move or stay in equilibrium.

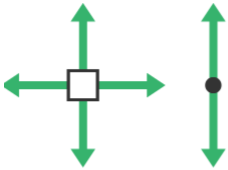
- **Key features:**
  - Arrows show direction and relative strength of each force.
  - Forces are labeled: F_{grav}, F_{norm}, F_{fric}, etc.
  - Moments/torques shown as curved arrows.
  - A coordinate system must be clearly defined.
- Assumptions are often made (e.g., neglecting air resistance unless stated).

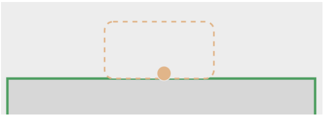


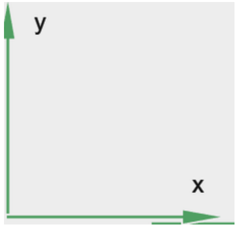


How to draw FBDs?

- **Step 1: Isolate object**
  Identify the body and draw the boundary. Simplify the object — represent it as a box, dot, or line.
- **Step 2: Contact Surfaces**
  Identify all surfaces in contact with the object.
  - Mark contact points with dots.
  - Each dot implies a contact force will act there.
- **Step 3: Coordinate system**
  Define a coordinate system.
  - Choose orientation (e.g., standard or slope-aligned).
  - Clearly mark positive x and y directions.



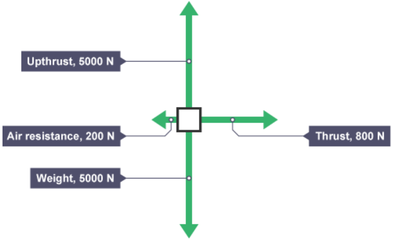


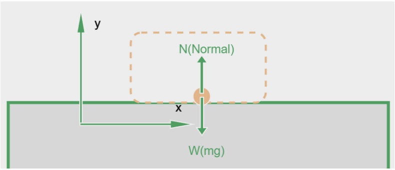


How to draw FBDs?

Identify forces

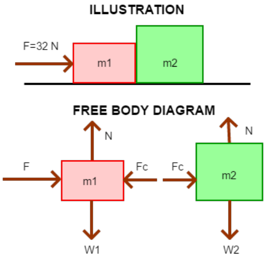
- **Step 4:** Add **contact forces** at the contact points:
  - F_{norm}: perpendicular to surface
  - F_{fric}: along the surface, opposes motion
  - F_{tens}, F_{spring}, F_{app} if applicable
- **Step 5:** Add **non-contact forces**:
  - $F_{\text{grav}} = mg$ (always vertically downward)
  - Add electric or magnetic forces if mentioned
- Use arrows pointing **towards the object**, label clearly.

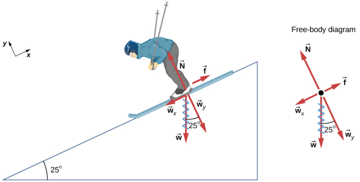




Step 6 & 7: Resolve Components + Multiple Bodies

- **Step 6:** Resolve angled forces into components:
    - Use trig: $F_x = F\cos\theta$, $F_y = F\sin\theta$
    - Replace original vector with its components
- **Step 7:** For multi-body systems:
    - Draw one FBD per object
    - Show interaction forces (equal and opposite)
    - Keep diagrams clean and separate



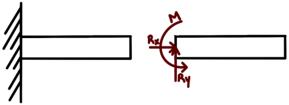


Support Reactions in FBDs – Introduction

- Structures like beams and frames are held by supports that resist motion.
- **Supports exert reaction forces and moments** to prevent unwanted translation and rotation.
- In FBDs, we **replace supports** with the reactions they apply.
- **Procedure:**
  - Isolate the body and traverse its perimeter.
  - At each support point, add arrows for forces and/or moments.
  - If direction is unknown, assume one.
  - Label each force or couple clearly.
- **Forces** prevent linear motion; **Moments** prevent rotation.

Example: Beam and Support Model – Grand Canyon Skywalk

- Imagine a beam extending from a wall: how much load can it handle?
- This depends on the **type of support** at the wall.
- **Example: Grand Canyon Skywalk**
  - Tourists walk on a transparent bridge above a canyon.
  - To ensure safety, the structure must resist all movements.
  - It is modeled as a **cantilever beam with fixed support**.
  - The support restricts:
    - Horizontal displacement
    - Vertical displacement
    - Rotation
- This abstraction allows 2D mechanical analysis using FBDs.



Fixed beam (cantilever) supporting the Skywalk

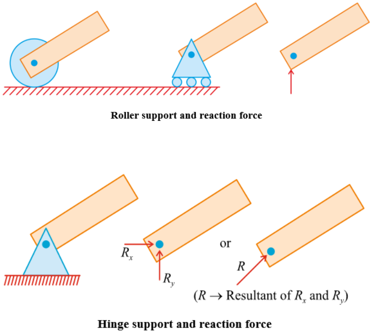
Types of Supports – Roller and Hinge

1. Roller Support

- Equivalent to a frictionless surface.
- Allows horizontal motion.
- Restricts vertical motion.
- Provides a single vertical reaction force.

2. Hinge (Pin) Support

- Allows rotation but not translation.
- Restricts horizontal and vertical movement.
- Provides two reaction forces: R_x and R_y.

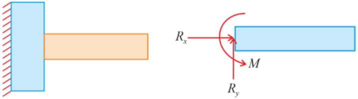


Support reactions for roller and hinge

Fixed Support (Built-in Constraint)

3. Fixed Support

- Also known as a clamped or built-in support.
- Restricts all motion: both translation and rotation.
- Provides three reactions:
  - R_x – horizontal force
  - R_y – vertical force
  - M – moment resisting rotation
- Common in wall-mounted structures, robotic arms, bridges.
- Converts real-world complexity into a solvable static model.



Fixed support and its reactions

All reactions at a fixed support

Two-Dimensional Support Reactions

- In 2D statics, bodies can translate in x and y, and rotate about the out-of-plane z-axis.
- Supports resist motion by supplying appropriate **reaction forces and/or moments**.
- We do not draw the actual support in the FBD, but we show its **reaction effects**.
- **Example: Door hinge (top view)**:
  - Allows rotation around hinge axis.
  - Prevents translation in both x and y.
  - Modeled as two perpendicular reaction forces, no moment.
- All 2D supports can be modeled by identifying which motions are constrained and drawing equivalent reactions.

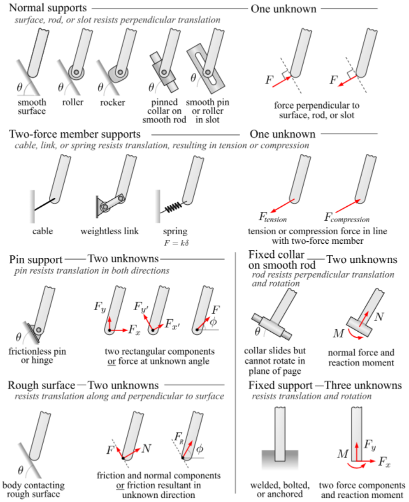
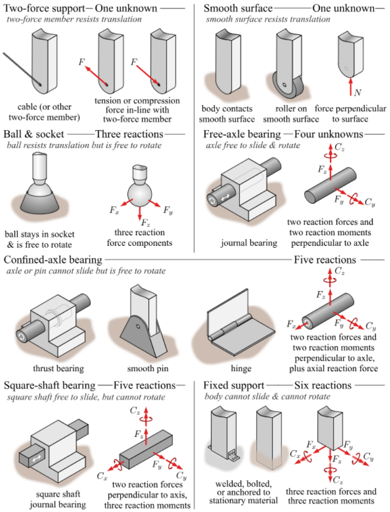


Image source:
`https://engineeringstatics.org/Chapter_05-free-body-diagrams.html`

Three-Dimensional Support Reactions

- 3D bodies have six degrees of freedom:
  - Translation in x, y, z
  - Rotation about x, y, z
- Supports in 3D may supply up to **6 reactions**:
  - 3 forces: F_x, F_y, F_z
  - 3 moments: M_x, M_y, M_z
- In FBDs, we show only the **reaction effects**, not the physical support itself.
- Identify what motions are **blocked** by the support, and show appropriate arrows.
- The more constraints a support imposes, the more reaction components it provides.



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`https://engineeringstatics.org/Chapter_05-free-body-diagrams.html`

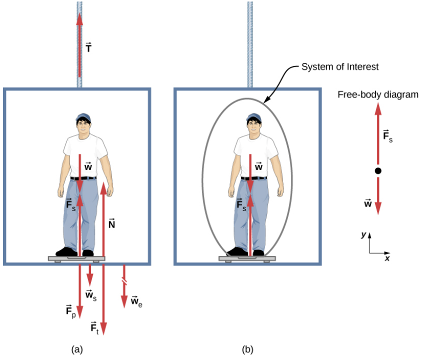
Example 1 – Scale Reading in an Elevator: Problem

Problem: A 75.0 kg man stands on a bathroom scale inside an elevator.

Determine the reading on the scale:

(a) When the elevator accelerates **upward** at $1.20\,\mathrm{m/s^2}$.

(b) When the elevator moves **upward at constant speed** of $1.00\,\mathrm{m/s}$.



Given:

- Mass of person: $m = 75.0\,\mathrm{kg}$
- Acceleration due to gravity: $g = 9.80\,\mathrm{m/s^2}$

Goal: Find the normal force F_s exerted by the scale, which is what the scale reads.

Image source:
https://phys.libretexts.org/Courses/Coalinga_College/Physical_Science_for_Educators

Example 1 – Scale Reading in an Elevator: Solution

Solution: Apply Newton's Second Law:

$$F_{\text{net}} = F_s - mg = ma \Rightarrow F_s = ma + mg$$

(a) Elevator accelerating upward:

$$F_s = (75.0)(1.20) + (75.0)(9.80)$$

$$F_s = 90.0 + 735 = \boxed{825\,\text{N}}$$

(b) Elevator moving at constant speed ($a = 0$):

$$F_s = (75.0)(0) + (75.0)(9.80) = \boxed{735\,\text{N}}$$

Interpretation:

- At constant speed or at rest: scale reads true weight.
- During upward acceleration: scale reads more than weight (apparent weight increases).

Example 2 – Djedi Robot Climbing an Incline: Problem

Problem: A compact exploration robot of mass M_r is designed to climb an inclined tunnel at angle θ_a inside a pyramid.

To successfully ascend the incline, it must overcome:

- Gravitational pull down the slope (F_g)
- Frictional drag due to a tether (F_c)
- Its own frictional limits on contact surface ($F_p \leq \mu_n F_r$)

Given:

- Robot mass: M_r, gravity: g
- Tether mass per meter: C_m, length: x_c, friction coeff: μ_c
- Surface friction coeff: μ_n

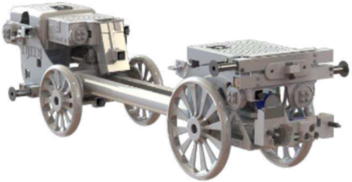


Fig. 1. Djedi Southern Shaft Rover.

Solution:

Gravitational Components:

$$F_r = M_r g \sin(\theta_a) \;\; and \;\; F_g = M_r g \cos(\theta_a)$$

Frictional Limit:

$$F_p \leq \mu_n F_r$$

Tether Drag Force:

$$F_c = C_m x_c g \left[\sin(\theta_a) + \mu_c \cos(\theta_a)\right]$$

Required Pulling Force:

$$F_p = M_r g \sin(\theta_a) + F_c$$

Condition: F_p must not exceed $\mu_n F_r$ to maintain

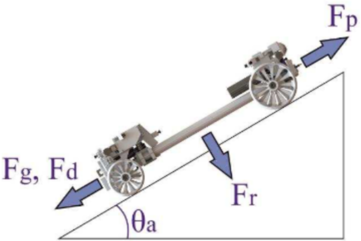


Fig. 8. Free body diagram of robot on an incline.

Example 2 – Djedi Robot Climbing an Incline

Why the Free-Body Diagram (FBD) Matters

The Free-Body Diagram helps us isolate the robot and represent all external forces acting on it—gravitational components, pulling force, normal force, and cable drag.

It allows us to:

- Resolve forces parallel and perpendicular to the incline.
- Quantify the total resistance the robot must overcome.
- Evaluate whether the pulling force stays within the frictional limit.

Without the FBD, the interplay of these forces would be difficult to track and analyze systematically.

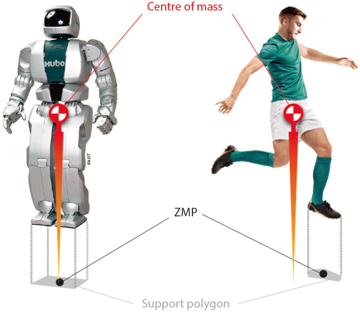
(Adapted from: Section 5, "Exploration Robots for Harsh Environments and Safety") —

https://www.researchgate.net/publication/282314003

Part B- Center of Mass (COM)

What is Center of Mass (COM)?

- For every system and at every instant in time, there exists a unique location that is the average position of the system's mass.
- This point is called the **center of mass** (COM), also known as *cm*, *c.o.m.*, *G*, or *c.g.*
- COM may or may not lie inside the object (e.g., boomerang or donut).
- Complex multiparticle systems like humans or robots can be simplified by tracking the COM instead of each part.
- Even with changing shape or posture, COM provides a single representative point of motion.



Example COM Illustration

Insight: Motion of the entire body can be approximated by following the trajectory of the COM.

Center of Mass vs. Center of Gravity

Center of Mass (COM)

- Geometric/mass-based concept
- Exists in any environment (even space)
- Represents average position of total mass

On Earth: COM $\approx$ COG

Center of Gravity (COG)

- Based on gravitational force
- Defined only in a gravitational field
- Location may shift in non-uniform gravity

COM Equations – Discrete and Continuous Systems

1. Discrete System:

$$\vec{r}_{\text{COM}} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

- Each particle's position is weighted by its mass.
- Useful for robots with known component masses (e.g., motor, battery).

2. Continuous Mass Distribution:

$$\vec{r}_{\text{COM}} = \frac{1}{M_{\text{tot}}} \int \vec{r}\, dm \quad \text{where} \quad dm = \begin{cases} \rho\, dV & \text{(volume)} \\ \rho\, dA & \text{(area)} \\ \rho\, ds & \text{(curve)} \end{cases}$$

- Used when mass is spread continuously along volume, surface, or line.

Example 1: System of Two Point Masses

Problem: Two point masses m_1 and m_2 are located at positions $\vec{r}_1$ and $\vec{r}_2$, respectively.
Find the position of the center of mass of the system.

- Center of mass:

$$\vec{r}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

- Rewriting:

$$\vec{r}_{\text{cm}} = \vec{r}_1 + \frac{m_2}{m_1 + m_2}(\vec{r}_2 - \vec{r}_1)$$

Interpretation:

- COM lies along the line connecting $\vec{r}_1$ and $\vec{r}_2$
- Closer to the larger mass

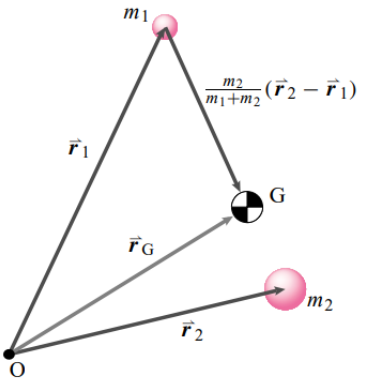


Figure 2.68: Center of mass of a system consisting of two points.

Example 2: Center of Mass of a Uniform Rod

Problem: A uniform rod has: Length $L = 3\,\mathrm{m}$ and Mass $m = 7\,\mathrm{kg}$. **Find the center of mass of the rod.**

- COM definition:

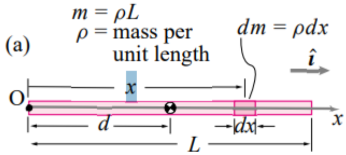
$$\vec{r}_{\mathrm{cm}} = \frac{1}{M}\int x\, dm$$

- Use linear mass element: $dm = \rho\, dx$
- Substitute and integrate:

$$\vec{r}_{\mathrm{cm}} = \frac{1}{m}\int_0^L x\rho\, dx = \frac{\rho}{m}\cdot\frac{L^2}{2}$$

- Since $\rho = \frac{m}{L}$:

$$\vec{r}_{\mathrm{cm}} = \frac{L}{2}\hat{i}$$



Answer: COM is at the midpoint — 1.5 m from either end.

Example 3: Change in COM of Human Body with Arm Movement (Part 1)

Problem: A person has total mass $M = 90\,\text{kg}$ and each arm has mass $m = 15\,\text{kg}$. The center of mass of each arm shifts vertically by $d = 18\,\text{cm} = 0.18\,\text{m}$ upward when raised.

Goal: Find how much the overall center of mass (COM) of the body rises when both arms are raised from down to up.

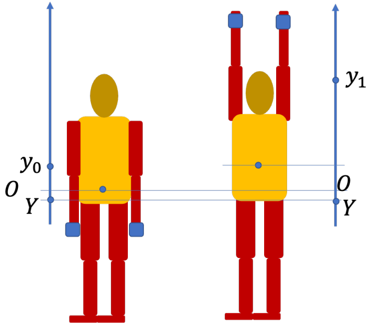
Coordinate Setup: Let $y = 0$ be the initial COM of the body (arms down). Let y_0 = COM of each arm when down, y_1 when up. Let Y = COM of the rest of the body (fixed).

Step 1: COM when arms are down

$$My_{\text{cm}} = 2my_0 + (M - 2m)Y, \quad y_{\text{cm}} = 0 \Rightarrow 0 = 2my_0 + (M - 2m)Y \tag{1}$$

Step 2: COM when arms are up

$$Mh = 2my_1 + (M - 2m)Y \tag{2}$$



Arms Down vs Arms Up – COM shift

Example: Change in COM of Human Body with Arm Movement (Part 2)

Step 3: Subtract Eq. (1) from Eq. (2):

$$Mh = 2m(y_1 - y_0) = 2md$$

Step 4: Plug in given values in:

$$\Rightarrow h = \frac{4m}{M} \cdot d$$

$$m = 15\,\text{kg}, \quad M = 90\,\text{kg}, \quad d = 0.18\,\text{m}$$

$$h = \frac{4 \cdot 15}{90} = 0.12\,\text{m}$$

Final Answer: The center of mass of the person rises by $\boxed{12\,\text{cm}}$ when both arms are raised.