Session 4: Motion and Statics

Part A- Motion: Linear and Angular
Part B- Statics

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Session 3 Recap & Reflect: What Did We Learn?

Session 3 Recap & Reflect

Frame Transformations Robots operate in a world of motion and orientation. To describe and control this motion, we need transformations.

- Coordinate transformations allow us to express points in different reference frames.
- Two fundamental components:
  - **Translation** – changes position.
  - **Rotation** – changes orientation.
- Together, they define the **Pose**:

$$\text{Pose} = \text{Position} + \text{Orientation}$$

- We use **Homogeneous Transformation Matrices (HTMs)** to combine both in a single 4×4 matrix.

Basic Translation in 2D and 3D

- **Translation** refers to moving a point or frame from one position to another without changing its orientation.
- It is the simplest form of transformation and a foundation for:
  - Motion planning
  - Coordinate frame conversions

1. Translation in 2D:

$$P = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{d} = \begin{bmatrix} dx \\ dy \end{bmatrix}, \quad P' = P + \vec{d} = \begin{bmatrix} x + dx \\ y + dy \end{bmatrix}$$

2. Translation in 3D:

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \vec{d} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}, \quad P' = P + \vec{d} = \begin{bmatrix} x + dx \\ y + dy \\ z + dz \end{bmatrix}$$

Rotation in 2D and 3D – Final Forms & Properties Rotation changes the orientation

of a point or frame without altering its position.
It is represented using a **rotation matrix** that depends on the axis and the angle of rotation.

1. **Rotation in 2D (about the origin):**

$$\vec{p}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad \text{where } R(\theta) \in \mathbb{R}^{2\times 2}$$

2. **Rotation in 3D (about Z-axis – Yaw):**

$$\vec{p}_{\text{new}} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{bmatrix} \quad \text{where } R \in \mathbb{R}^{3\times 3}$$

3. Rotation in 3D (other principal axes):

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad \text{(Pitch)} \qquad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad \text{(Roll)}$$

4. Properties of Rotation Matrices:

- Orthogonal: $R^\top R = I$
- Determinant: $\det(R) = 1$
- Preserve distances and angles

Homogeneous Transformation Matrix What is it?

A homogeneous transformation matrix combines both rotation and translation into a single 4×4 matrix. It allows transformation of points or frames using matrix multiplication — avoiding the need to apply rotation and translation separately.

Structure:

$$T = \begin{bmatrix} R & \vec{d} \\ \mathbf{0}^\top & 1 \end{bmatrix} \quad \text{where } R \in \mathbb{R}^{3\times 3}, \quad \vec{d} \in \mathbb{R}^{3\times 1}$$

Component Form:

$$T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transforming a Point in Homogeneous Coordinates:

$$\vec{p}_h = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \Rightarrow \quad \vec{p}'_h = T \cdot \vec{p}_h = \begin{bmatrix} r_{11}x + r_{12}y + r_{13}z + d_x \\ r_{21}x + r_{22}y + r_{23}z + d_y \\ r_{31}x + r_{32}y + r_{33}z + d_z \\ 1 \end{bmatrix}$$

Compact Form:

$$\vec{p}'_h = \begin{bmatrix} R \cdot \vec{p} + \vec{d} \\ 1 \end{bmatrix} \quad \text{where } \vec{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Chaining Transformations Using Homogeneous Matrices

Key Idea: To find the position of a point in a distant frame, we chain multiple transformation matrices using matrix multiplication.

Scenario:
Suppose we have three reference frames: A, B, and C.

- T_{AB}: transformation from frame A to frame B
- T_{BC}: transformation from frame B to frame C

To transform a point $\vec{x}_A$ from frame A to frame C:

$$\vec{x}_C = T_{BC} \cdot T_{AB} \cdot \vec{x}_A$$

- First apply T_{AB}: maps from A to B, Then apply T_{BC}: maps from B to C

General Case (Longer Chains):

$$\vec{x}_E = T_{DE} \cdot T_{CD} \cdot T_{BC} \cdot T_{AB} \cdot \vec{x}_A$$

Session 3 Recap Quiz

Recap Quiz

1. Why do we use homogeneous coordinates for transformation in 3D space?
   A. They simplify linear motion equations
   B. They remove the need for trigonometric functions
   C. They allow combining rotation and translation into one matrix
   D. They help visualize 2D shapes

2. If a rotation matrix is applied to a point but the translation vector is zero, what kind of motion is it?
   A. Pure translation
   B. Rigid body transformation
   C. Pure rotation
   D. Scaling

Recap Quiz (contd.)

3. In robotics, the rotation of a drone's nose up and down is best described by which of the following?
   A. Roll
   B. Pitch
   C. Yaw
   D. Elevation

4. When using homogeneous matrices, why is the order of matrix multiplication critical in chaining transformations?
   A. Matrices are always commutative
   B. Each transformation depends on the previous frame's orientation and position
   C. It doesn't matter — order has no effect
   D. Matrices cancel each other out if reversed

Recap Quiz (contd.)

5. Suppose T_{AB} maps from base to elbow, and T_{BC} maps from elbow to wrist. What is T_{AC}?
   A. Direct mapping from wrist to elbow
   B. Inverse mapping from wrist to base
   C. Compound transformation from base to wrist
   D. Cannot be computed without joint angles

6. Which of these rotation matrices rotates a vector around the Y-axis?
   A. $R_z(\theta)$
   B. $R_y(\theta)$
   C. $R_x(\theta)$
   D. $R_{\text{pitch}}(\theta)$

Recap Quiz (contd.)

7. A mobile robot starts at the origin $(0, 0)$ and is initially facing East. It moves 2 meters forward and then rotates 90° to the right. What is its final position and heading relative to the original frame?
   A. Position: $(2, 0)$, Heading: East
   B. Position: $(2, 0)$, Heading: South
   C. Position: $(0, 2)$, Heading: North
   D. Position: $(0, -2)$, Heading: West

8. A camera mounted on a robot observes an object at (3, 0, 0) in its local frame. The camera is positioned at (0, 0, 5) and rotated 180° around the Z-axis. Where is the object in world coordinates?
   A. (3, 0, 5)
   B. (–3, 0, 5)
   C. (–3, 0, 5)
   D. (0, –3, 5)

Quiz Answer Key with Explanations

- **Q1 Answer: C** Allow both rotation and translation to be combined into a single matrix multiplication.
- **Q2 Answer: C** If there's no translation, applying a rotation matrix results in a pure rotation about the origin.
- **Q3 Answer: B (Pitch)** Refers to rotation around the Y-axis — it tilts the drone's nose up or down, just like nodding your head.
- **Q4 Answer: B** The order matters because each transformation applies relative to the frame resulting from the previous transformation.
- **Q5 Answer: C** $T_{AC} = T_{AB} \cdot T_{BC}$ maps from base (A) to wrist (C), by composing the transformations between joints.
- **Q6 Answer: B and D** $R_y(\theta)$ and Pitch both describe rotation around the Y-axis. They are equivalent terms in robotics and aerospace conventions.

Quiz Answer Key with Explanations (contd.)

Q7 Answer: B — (2, 0), now facing South direction (270°)(-Y)

Step 1: Initial Position and Heading

The robot starts at the origin (0, 0), facing +X (East).

Step 2: Move Forward 2 meters

This puts it at position (2, 0).

Step 3: Rotate 90° Right (Clockwise)

Now the robot faces –Y (South), which is 270° from the original orientation.

Final Result:

Position: (2, 0), Heading: –Y axis (South)

Quiz Answer Key with Explanations (contd.)

Q8 Answer: C — (–3, 0, 5)

Step 1: Camera observes object at $(3, 0, 0)$ in local frame.

Step 2: Rotation Matrix — 180° about Z-axis:

$$R_z(180^\circ) = \begin{bmatrix} \cos 180^\circ & -\sin 180^\circ & 0 \\ \sin 180^\circ & \cos 180^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: Translation Vector: $\vec{d} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

Quiz Answer Key with Explanations (contd.)

Step 4: Full Transformation Matrix:

$$T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 5: Multiply with point:

$$\vec{p}_{\text{world}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

Final Result: The object is at $(-3, 0, 5)$ in world coordinates.

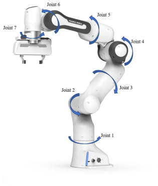
Part A- Motion: Linear and Angular

Why Study Rotational Motion in Robotics?

Robots Don't Just Move — They Rotate

Robotic systems frequently rely on **rotational motion** to perform tasks:

- **Robot arms** rotate their joints to position tools or grippers.
- **Wheeled robots** rotate wheels to drive, steer, and navigate.
- **Cameras and sensors** pan and tilt using rotational actuators.



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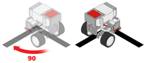

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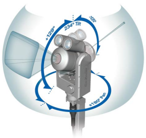

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Image Sources:

1. https://https://www.researchgate.net/publication/361659008_RoboGroove_Creating_Fluid_Motion_for_Dancing_Robotic_Arms/
2. http://cmra.rec.ri.cmu.edu/products/robotc_ev3_curriculum/lesson/1-3Turning3.html
3. https://www.fwbbo.shop/?ggcid=5335250

Linear vs Angular Motion: Why Compare Them?

Robotic systems often involve both types of motion.

Why Compare Them?

- Linear motion: movement along a straight path.
- Angular motion: rotation about a fixed axis.

Where You'll See Both in Robotics

Scenario	Linear	Angular
Robot arm reaching out	End-effector translation	Joint rotation (θ, ω, τ)
Wheeled robot navigating	Forward motion (v)	Wheel rotation(ω)
Drone tilting in air	Lift or descent (a)	Pitch/Roll/Yaw (α, τ)
Humanoid walking	Step forward (v)	Hip/knee joint rotation

Linear vs Angular Motion: Quantity Mapping

Core Quantities in Motion

Linear	**Angular**
Position x	Angular position θ
Displacement Δx	Angular displacement $\Delta\theta$
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Force F	Torque τ
Mass m	Moment of Inertia I

Each angular quantity mirrors a linear counterpart — this symmetry helps us transfer intuition and apply both in robotic control and dynamics.

What is a Rigid Body?

Definition and Significance

- A **rigid body** is a theoretical object made up of particles that remain at fixed distances from one another, regardless of motion or applied forces.
- In an ideal scenario, it does not bend, stretch, or deform.
- It is often treated as a continuous distribution of mass

Why Use Rigid Body Models?

- All real-world objects are deformable to some extent.
- However, in many mechanical systems, the deformation is negligible and can be ignored.
- This assumption allows us to use simpler equations to describe how objects move and rotate.

In robotics, we idealize arms, wheels, and links as rigid bodies to simplify motion analysis and focus on position, velocity, and force — without worrying about internal deformation.

Measuring Angles: Degrees vs Radians

Measuring Angles: Degrees vs Radians

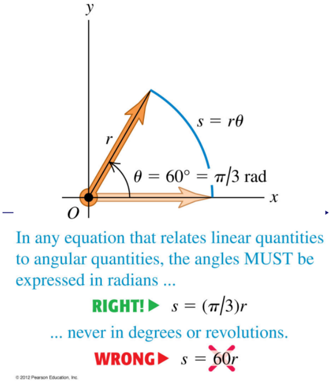
- We mostly come across **degrees** to measure angles — where a full circle is 360°.
- In advanced physics, and robotics, we often use **radians** instead of degrees.
- A **radian** is the standard unit for measuring angles in scientific contexts

What is a Radian?

- Defined as the angle subtended at the center of a circle when the arc length s equals the radius r.
- Formula:

$$\theta = \frac{s}{r}$$

  where: θ: angle in radians, s: arc length, and r: radius of the circle
- Radians are technically dimensionless, but we use "rad" for clarity.



Conversion Between Radians and Degrees

Converting Radians to Degrees

- The relationship between radians and degrees:

$$2\pi \text{ radians} = 360^\circ$$

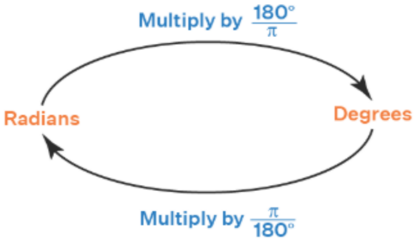
- From this, we derive:

$$1 \text{ radian} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

- Conversion formulas:

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$



Degrees Radians conversion

Linear Position x

What is motion? Motion is a change of position over time.

- **How do we represent position along a straight line?**
  - Define a starting point: the **origin**, where $x = 0$
  - Position is measured **relative to the origin**
  - **Direction matters**:
    - Positive: to the right or upward
    - Negative: to the left or downward
- **Time-dependent position**:
  - Position changes with time: $x(t)$
  - At $t = 0$, position doesn't have to be zero: $x(0) \neq 0$
- **Units**: Position is measured in **meters (m)**

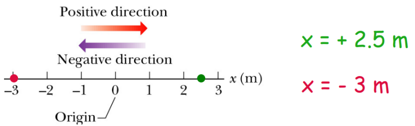


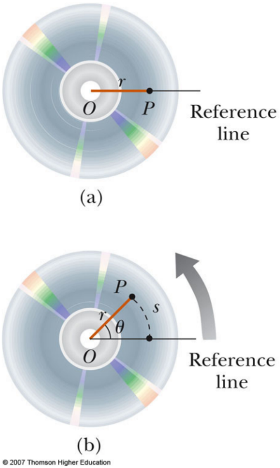
Illustration: Position along a straight line with origin

Defining Rotation and Angular Position

- Rotation occurs about a fixed **axis** — e.g., the center of a disc.
- Choose a **fixed reference line** (e.g., horizontal).
- Let point P be located at a fixed distance r from the axis.
- As the object rotates, point P traces an **arc** of length s.
- The **angle** subtended at the center is:

$$\theta = \frac{s}{r}$$

- The only changing coordinate of the point is θ — its **angular position**.
- **Units**: Angular position is measured in **radians (rad)**



Linear Displacement Δx

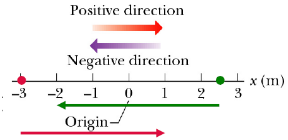
Definition and Properties

- Displacement is the change in position over time:

$$\Delta x = x(t_{\text{final}}) - x(t_{\text{initial}})$$

- **Vector quantity** — has both magnitude and direction.
- **Sign indicates direction:**

  $+\Delta x$: Motion to the right or upward, $-\Delta x$: Motion to the left or downward
- Units: meters (m).
- Different from distance — distance is scalar, always positive.

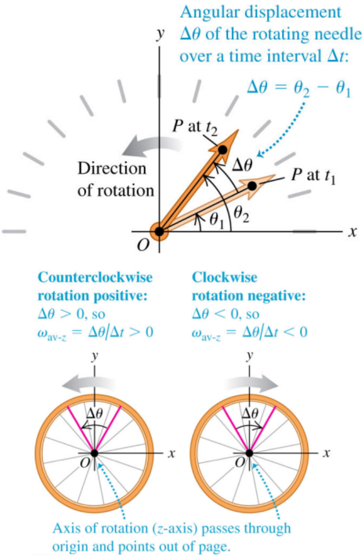


$x_1(t_1) = +2.5$ m
$x_2(t_2) = -2.0$ m
$\Delta x = -2.0$ m $- 2.5$ m $= -4.5$ m

$x_1(t_1) = -3.0$ m
$x_2(t_2) = +1.0$ m
$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m

Definition and Properties

- Angular displacement is the change in angular position:
$$\Delta\theta = \theta_{\text{final}} - \theta_{\text{initial}}$$
- **Vector quantity** — describes both how much and in which direction the rotation occurred.
- **Sign indicates rotation direction:**
  - $+\Delta\theta$: Counterclockwise
  - $-\Delta\theta$: Clockwise
- Units: radians (rad)



Linear Velocity v

- Notation: v **Unit:** meters per second (m/s)
- Velocity is the **rate of change of position** with respect to time.
- It is a **vector quantity** — it has both magnitude and direction.

Average vs Instantaneous Velocity

- **Average velocity:**

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- **Average speed:**

$$s_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}$$

- Speed is a **scalar**; velocity is a **vector**
- **Instantaneous velocity:**

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Angular Velocity ω

What is Angular Velocity?

- Notation: ω **Unit:** radians per second (rad/s)
- Angular velocity describes how fast an object is **rotating**.
- It is the **rate of change of angular position** over time.
- **Average angular velocity:**

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

- **Instantaneous angular velocity:**

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Angular velocity has the **same structure** as linear velocity — it's just rotational.

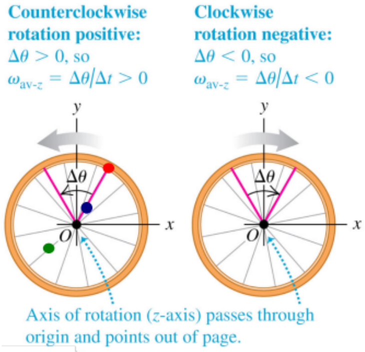
Angular Velocity ω

Sign Convention

- $\omega > 0$: **Counterclockwise (CCW)** rotation
- $\omega < 0$: **Clockwise (CW)** rotation

Rigid Body Property

- In a rotating **rigid body**, all points share the same angular velocity ω
- But linear speed depends on radius: $v = r\omega$
- Points farther from the axis move faster in linear terms



Linear Acceleration – a

- **What is Acceleration?**
  - Acceleration is the rate of change of velocity with respect to time.
  - Vector quantity: has both magnitude and direction.
  - SI Unit: m/s^2
- **Average Acceleration:**

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- **Instantaneous Acceleration:**

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- **Key Notes:**
  - $a > 0$: Speeding up
  - $a < 0$: Slowing down (deceleration)
  - $a = 0$: Constant velocity (no acceleration)

Angular Acceleration – α

- **What is Angular Acceleration?**
  - The rate at which angular velocity changes over time.
  - Vector quantity with both magnitude and direction.
  - SI Unit: rad/s^2
- **Average Angular Acceleration:**

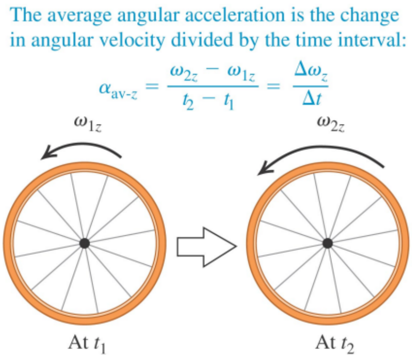
$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

- **Instantaneous Angular Acceleration:**

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Angular Acceleration – α

- **Sign Convention:**
  - $\alpha > 0$:
    - Rotating counterclockwise (CCW) and speeding up
    - Rotating clockwise (CW) and slowing down
  - $\alpha < 0$:
    - Rotating clockwise (CW) and speeding up
    - Rotating counterclockwise (CCW) and slowing down
- **Rigid Body Insight:**
  - All points on a rigid body share the same angular acceleration α.
  - Linear acceleration at a point is related by: $a = r\alpha$



Linking Angular and Linear Motion

- **Every point on a rotating object has the same:**
  - Angular displacement θ
  - Angular velocity ω
  - Angular acceleration α
- **But not the same linear quantities:**
  - Displacement $s = r\theta$
  - Velocity $v = r\omega$
  - Acceleration $a = r\alpha$
- These relations connect rotational motion to the linear motion of points located at a distance r from the axis.

Velocity Comparison

The linear velocity is always tangent to the circular path -called the tangential velocity.

A point on the perimeter of a rotating disk moves along an arc (displacement):

- From arc length:

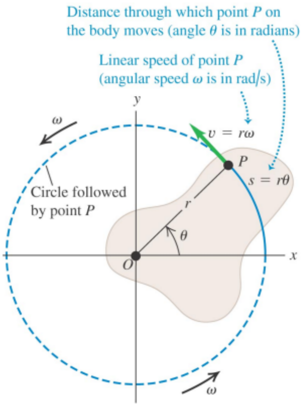
$$\Delta s = r \cdot \Delta\theta$$

- Rearranged:

$$\Delta\theta = \frac{\Delta s}{r}$$

- Divide by time:

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{r \cdot \Delta t} \Rightarrow \omega = \frac{v}{r} \; or, \; v = r\omega$$



- The tangential acceleration is the rate of change of tangential velocity:

$$a_t = \frac{\Delta v}{\Delta t}$$

- From earlier relation:

$$v = r\omega \Rightarrow \Delta v = r\Delta\omega$$

- Substitute in acceleration formula:

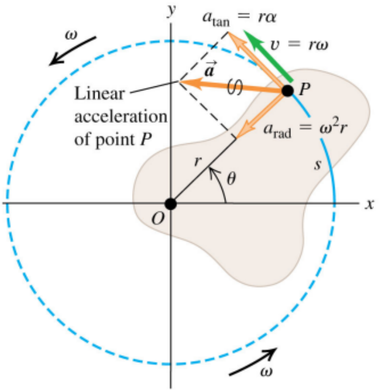
$$a_t = \frac{r\Delta\omega}{\Delta t} = r \cdot \frac{\Delta\omega}{\Delta t}$$

- Hence:

$$a_t = r\alpha$$

Radial and tangential acceleration components:

- $a_{rad} = \omega^2 r$ is point P's centripetal acceleration.
- $a_{tan} = r\alpha$ means that P's rotation is speeding up (the body has angular acceleration).

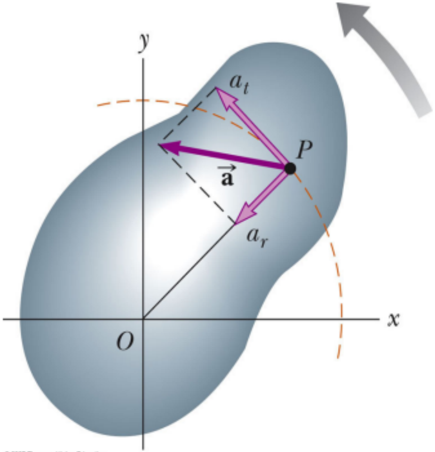


Centripetal Acceleration

- An object moving in a circle experiences an inward acceleration:

$$a_r = \frac{v^2}{r} = \omega^2 r$$

- Direction: always toward the center of rotation.
- Present even if speed is constant (because direction changes).



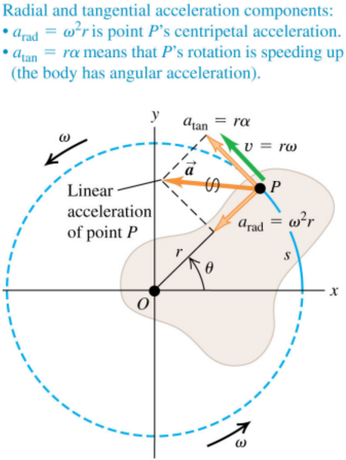
Resultant Acceleration

- Two components:
  - Tangential acceleration $a_t = r\alpha$
  - Radial (centripetal) acceleration

    $a_r = \omega^2 r$

- Resultant:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (\omega^2 r)^2} = r\sqrt{\alpha^2 + \omega^2}$$



Source for all images:
`https://web.njit.edu/~binchen/phys111/LectureNotes/Physics111_lecture10.pdf`

Mass m – Linear Inertia

- Mass is a measure of the amount of matter in an object.
- More importantly in mechanics, it quantifies the resistance to linear acceleration.
- Appears in Newton's Second Law:

$$F = ma$$

- Scalar quantity, measured in kilograms (kg).
- Larger mass $\Rightarrow$ greater resistance to change in velocity.

Moment of Inertia I – Rotational Inertia

- Analogous to mass in rotational motion.
- It quantifies resistance to angular acceleration.
- For a point mass:

$$I = mr^2$$

- m is mass, r is the perpendicular distance from the axis of rotation.
- Unit: $\mathrm{kg} \cdot \mathrm{m}^2$
- Depends on both mass and its distribution about the axis.

Moment of Inertia – Composite Objects

- For a system of point masses:

$$I = \sum_{i=1}^{n} m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$$

- Where:
  - m_i: mass of the ith particle
  - r_i: perpendicular distance from the axis of rotation
- The farther a mass is from the axis, the more it contributes to I.

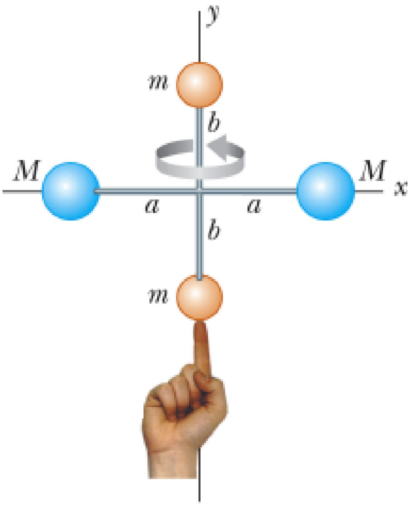
Example: Baton with 4 Masses

System Description:

- A baton lies in the x–z plane
- Two masses M are placed at $x = \pm a$.
- Two masses m are placed at $z = \pm b$.
- Rotation axis is along the y-axis.
- Moment of inertia about y-axis:

$$I_y = 2Ma^2 + 2m \cdot 0^2 = 2Ma^2$$

- Only the masses along the x-axis contribute.



Moment of Inertia – Extended (Continuous) Bodies

- Real-world objects are continuous bodies, not point masses.
- Need to sum over all infinitesimal mass elements.
- Transition from discrete to continuous:

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 \, dm$$

- Each Δm_i becomes an infinitesimal dm.
- **Summation turns into an integral**.

Computing Moment of Inertia – Integration

- To evaluate I for an extended body:
  1. Choose a coordinate system.
  2. Express r as a function of position, $r(x)$.
  3. Express the infinitesimal mass element as:

$$dm = \rho(x)\, dV$$

  4. Substitute and integrate:

$$I = \int r(x)^2\, \rho(x)\, dV$$

- $\rho(x)$: linear, area, or volume density depending on geometry.

Moment of Inertia – Common Shapes Quick Guide

- These formulas depend on the object's shape, mass, and how the axis of rotation is placed.